Instruction

Prerequisite Skills

This lesson requires the use of the following skills:

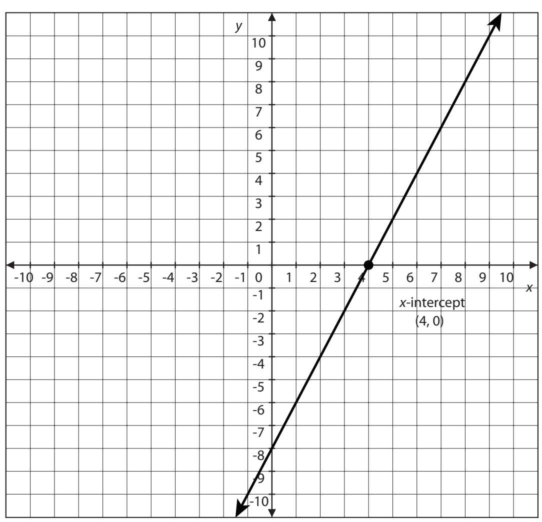
- graphing a linear function from a table or equation
- graphing an exponential function from a table or equation
- having knowledge of function notation, domain, and independent and dependent variables

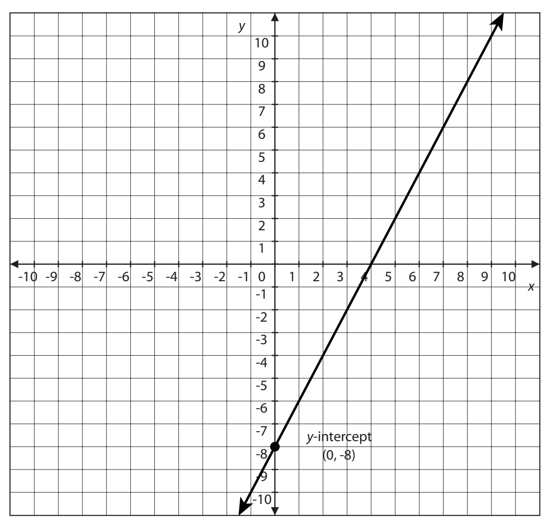
Introduction

Real-world contexts that have two variables can be represented in a table or graphed on a coordinate plane. There are many characteristics of functions and their graphs that can provide a great deal of information. These characteristics can be analyzed and the real-world context can be better understood.

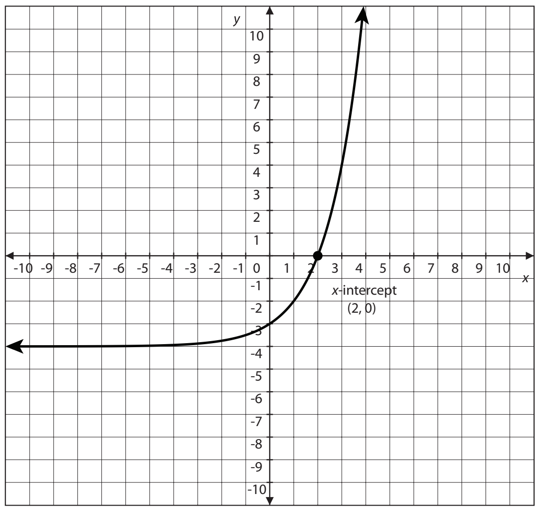
Key Concepts

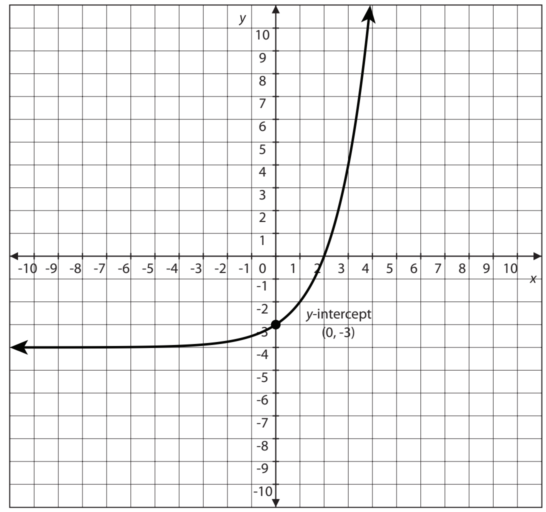
- One of the first characteristics of a graph that we can observe are the **intercepts**, where a function crosses the *x*-axis and *y*-axis.
  - The *y*-intercept is the point at which the graph crosses the *y*-axis, and is written as (0, *y*).
  - The ***x*-intercept** is the point at which the graph crosses the *x*-axis, and is written as (*x*, 0).



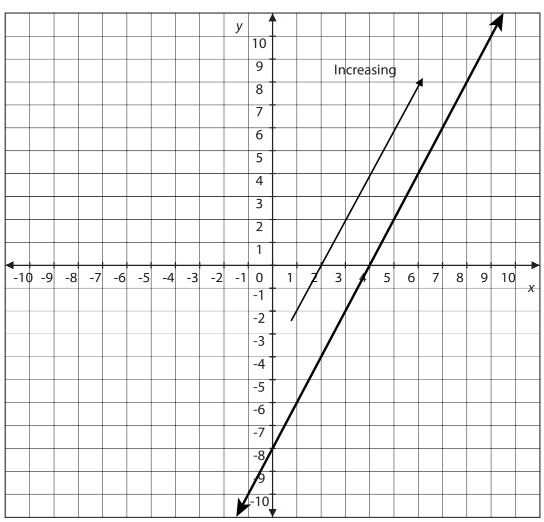


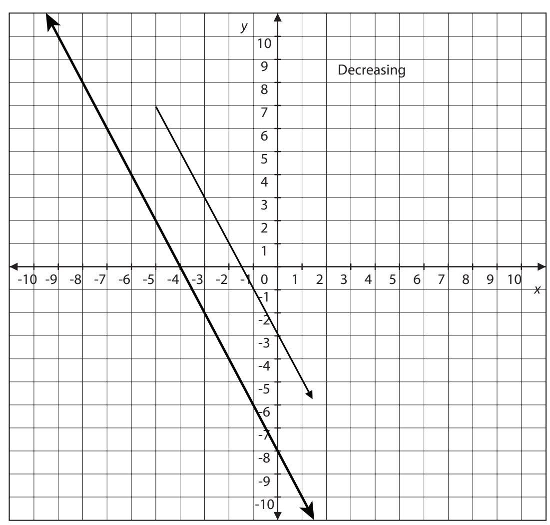
SRB pp. U2-84–U2-88

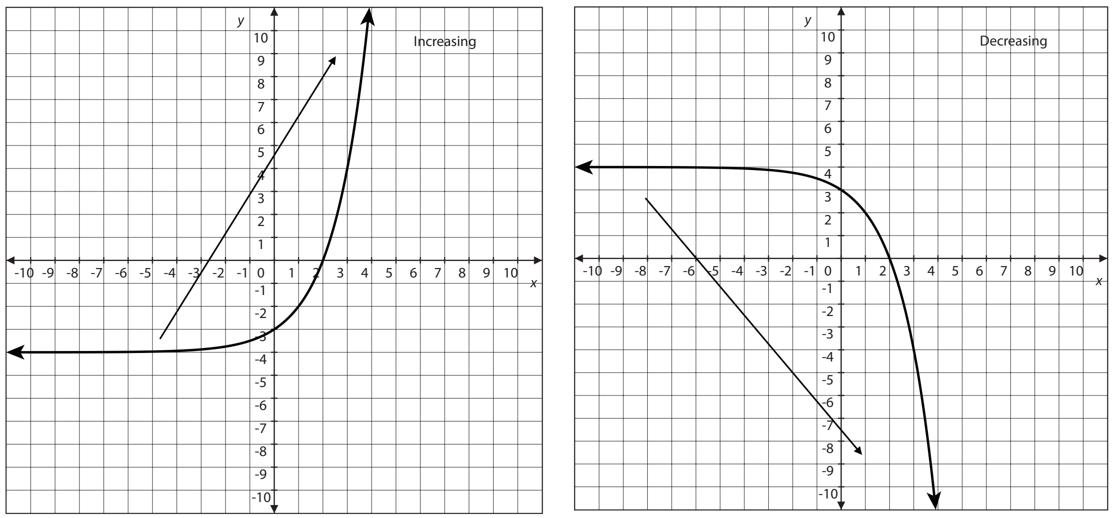




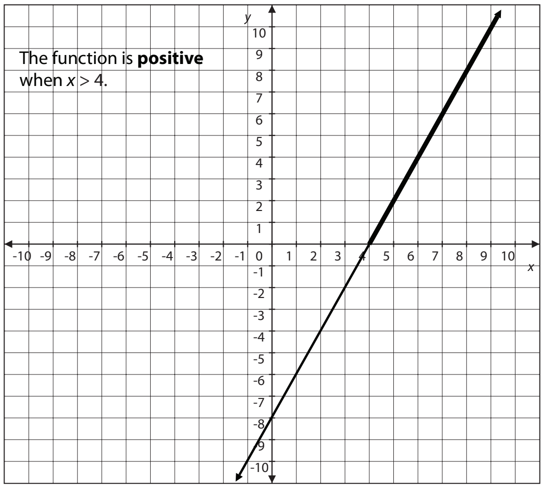
- Another characteristic of graphs that we can observe is whether the graph represents a function that is increasing or decreasing.
- When determining whether intervals are increasing or decreasing, focus just on the *y*-values.
- Begin by reading the graph from left to right and notice what happens to the graphed line. If the line goes up from left to right, then the function is increasing. If the line is going down from left to right, then the function is decreasing.
- A function is said to be constant if the graphed line is horizontal, neither rising nor falling.

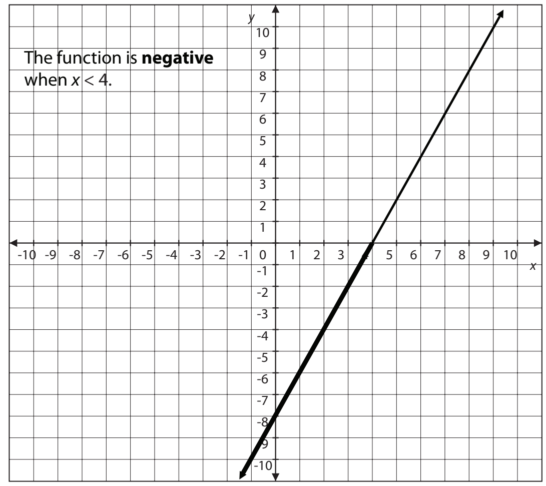


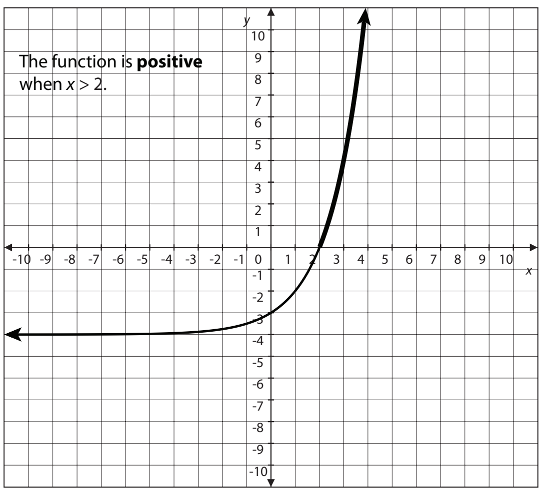


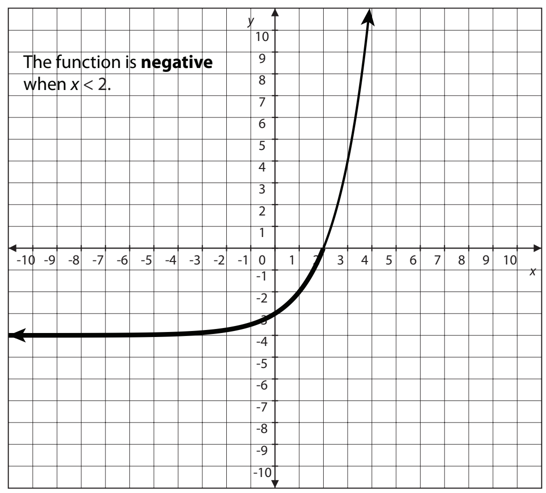


- An **interval** is a continuous series of values. (**Continuous** means "having no breaks.") A function is **positive** on an interval if the *y*-values are greater than zero for all *x*-values in that interval.
- A function is positive when its graph is above the *x*-axis.
- Begin by looking for the *x*-intercepts of the function.
- Write the *x*-values that are greater than zero using inequality notation.
- A function is **negative** on an interval if the *y*-values are less than zero for all *x*-values in that interval.
- The function is negative when its graph is below the *x*-axis.
- Again, look for the *x*-intercepts of the function.
- Write the *x*-values that are less than zero using inequality notation.

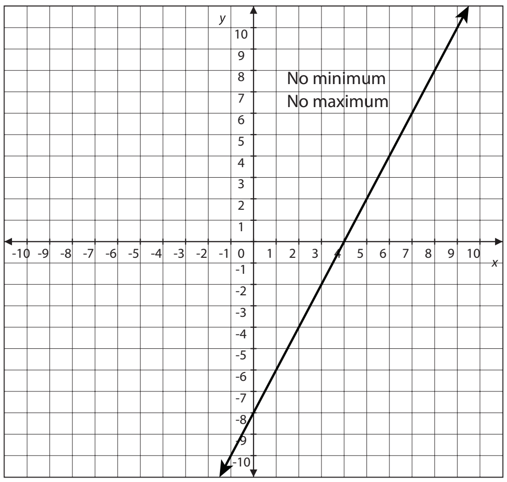


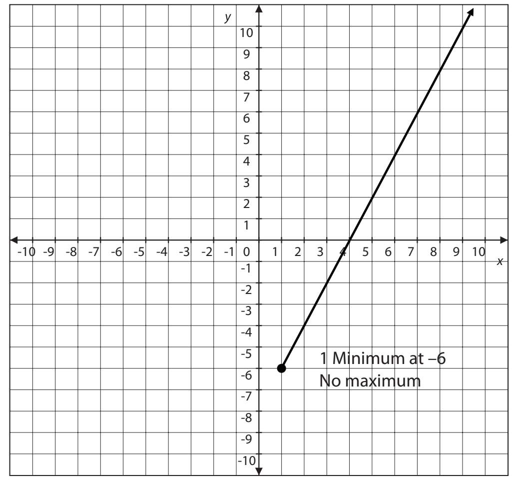






- Graphs may contain **extrema**, or minimum or maximum points.
- A **relative minimum** is the point that is the lowest, or the y-value that is the least for a particular interval of a function.
- A **relative maximum** is the point that is the highest, or the y-value that is the greatest for a particular interval of a function.
- Linear and exponential functions will only have a relative minimum or maximum if the domain is restricted.
- The **domain** of a function is the set of all inputs, or x-values of a function.
- Compare the following two graphs. The graph on the left is of the function $f(x) = 2x - 8$. The graph on the right is of the same function, but the domain is for $x \geq 1$. The minimum of the function is –6.



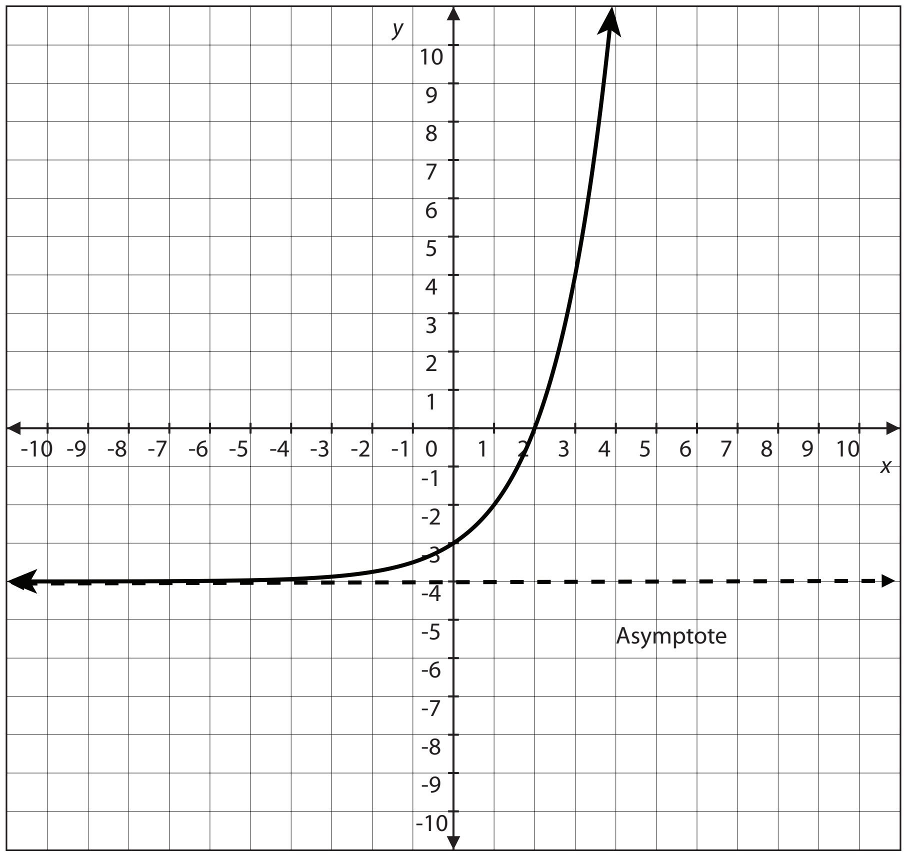


- Functions that represent real-world scenarios often include domain restrictions. For example, if we were to calculate the cost to download a number of e-books, we would not expect to see negative or fractional downloads as values for x.
- There are several ways to classify numbers. The following table lists the most commonly used classifications when defining domains.

Natural numbers	1, 2, 3, ...
Whole numbers	0, 1, 2, 3, ...
Integers	..., –3, –2, –1, 0, 1, 2, 3, ...
Rational numbers	numbers that can be written as $\frac{a}{b}$, where a and b are integers and $b \neq 0$; any number that can be written as a decimal that ends or repeats
Irrational numbers	numbers that cannot be written as $\frac{a}{b}$, where a and b are integers and $b \neq 0$; any number that cannot be written as a decimal that ends or repeats
Real numbers	the set of all rational and irrational numbers

- An exponential function in the form $f(x) = a^x$, where $a > 0$ and $a \neq 1$, has an **asymptote**, which is a line that a function gets closer and closer to as one of the variables increases or decreases without bound.

- The function in the following graph has a horizontal asymptote at $y = -4$.
- It may appear as though the graphed line touches $y = -4$, but it never does.



- Fairly accurate representations of functions can be sketched using the key features we have just described.

Common Errors/Misconceptions

- believing that exponential functions will eventually touch or intersect an asymptote
- incorrectly identifying the type of function as either exponential or linear
- misidentifying key features on a graph
- incorrectly choosing the domain for a function