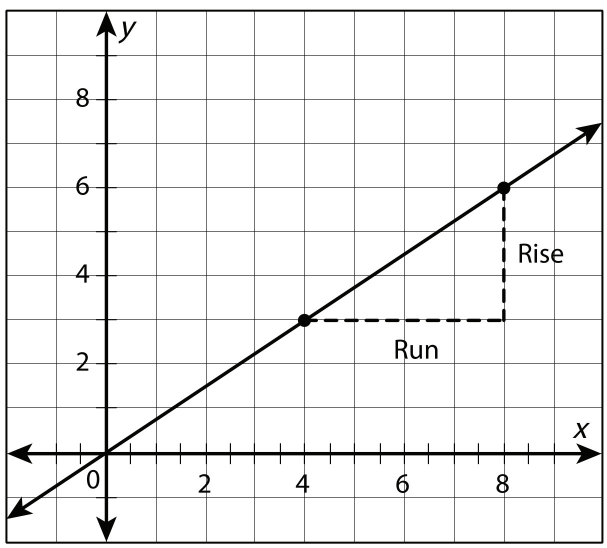
Skill 5: Finding the Slope or Rate of Change of Linear Functions

Introduction

A **proportional relationship** describes the relationship between two quantities that vary directly with one another. A few common proportional relationships that we encounter in our everyday lives include the speed a car travels (miles per hour), the amount of gas consumed on a road trip (gallons per mile), the amount of money earned at a job (dollars per hour), or the number of calories per serving of a favorite snack food (calories per serving). In all of these examples, each of the two quantities described varies directly with the other.

Key Concepts

- The quantities described by a proportional relationship are represented by a linear equation in the form $y = mx$, where m is the slope of the line that passes through the origin (0, 0).
- The **slope** of the graph of a linear equation is a measure of the rate of change of one variable with respect to another variable, and is defined by the ratio of the rise of the graph compared to the run.

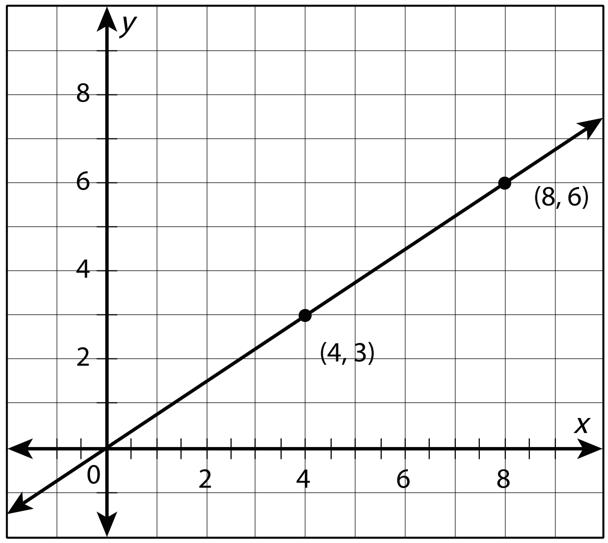


- Given two points on a line, (x_1, y_1) and (x_2, y_2), the slope is the ratio of the change in the y-values of the points (the rise) to the change in the corresponding x-values of the points (the run).

$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$$

- The first step in calculating the slope of a line is to choose two points on the line and label the coordinates of these points as (x_1, y_1) and (x_2, y_2). Then, the rate of change can be found by applying the slope formula. Reduce any fractions to ensure the slope is in simplest form.

- In the following graph, notice that two easily identifiable points on the line are (4, 3) and (8, 6).



- Let (x_1, y_1) be (4, 3) and (x_2, y_2) be (8, 6). Substitute these values into the slope formula and simplify to find the slope of the line.

$$\text{slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{(6)-(3)}{(8)-(4)} = \frac{3}{4}$$

- The given line has a rise of 3 units and a run of 4 units; therefore, the slope of the line is $\frac{3}{4}$.
- Note that if the assignment of (x_1, y_1) and (x_2, y_2) was switched in this example, the result would still be the same. For example, let (x_1, y_1) be (8, 6) and (x_2, y_2) be (4, 3). Substitute and simplify.

$$\text{slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{(3)-(6)}{(4)-(8)} = \frac{-3}{-4} = \frac{3}{4}$$

- The resulting slope is still $\frac{3}{4}$.
- Although it does not matter which point is (x_1, y_1) and which is (x_2, y_2), it *is* important to make sure that the order in which the variables are subtracted remains the same in the numerator and denominator. In other words, $\text{slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{y_1 - y_2}{x_1 - x_2}$; however, $\text{slope} \neq \frac{y_1 - y_2}{x_2 - x_1}$.
- The slope of an equation that describes a proportional relationship is also known as the **unit rate**, or the rate per one given unit.
- The calculation of slope can be extended beyond proportional relationships to that of linear equations of the form $y = mx + b$, where b is the y-intercept.