Math I

Unit 1: Expressions and Equations

"Our Place in the Universe"

Name:_______________________________

Prerequisite Skills

This lesson requires the use of the following skills:

- translating verbal expressions to algebraic expressions
- evaluating expressions following the order of operations

Introduction

Thoughts or feelings in language are often conveyed through expressions; however, mathematical ideas are conveyed through **algebraic expressions**. Algebraic expressions are mathematical statements that include numbers, operations, and variables to represent a number or quantity. **Variables** are letters used to represent values or unknown quantities that can change or vary. One example of an algebraic expression is $3x - 4$. Notice the variable, x.

Key Concepts

- Expressions are made up of **terms**. A term is a number, a variable, or the product of a number and variable(s). An addition or subtraction sign separates each term of an expression.
- In the expression $4x^2 + 3x + 7$, there are 3 terms: $4x^2$, $3x$, and 7.
- The **factors** of each term are the numbers or expressions that when multiplied produce a given product. In the example above, the factors of $4x^2$ are 4 and x^2. The factors of $3x$ are 3 and x.
- 4 is also known as the **coefficient** of the term $4x^2$. A coefficient is the number multiplied by a variable in an algebraic expression. The coefficient of $3x$ is 3.
- The term $4x^2$ also has an **exponent**. Exponents indicate the number of times a factor is being multiplied by itself. In this term, 2 is the exponent and indicates that x is multiplied by itself 2 times.
- Terms that do not contain a variable are called **constants** because the quantity does not change. In this example, 7 is a constant.

Expression	$4x^2 + 3x + 7$		
Terms	$4x^2$	$3x$	7
Factors	4 and x^2	3 and x	
Coefficients	4	3	
Constants			7

- Terms with the same variable raised to the same exponent are called **like terms**. In the example $5x + 3x - 9$, $5x$ and $3x$ are like terms. Like terms can be combined following the **order of operations** by evaluating grouping symbols, evaluating exponents, completing multiplication and division, and completing addition and subtraction from left to right. In this example, the sum of $5x$ and $3x$ is $8x$.

Common Errors/Misconceptions

- incorrectly following the order of operations
- incorrectly identifying like terms
- incorrectly combining terms involving subtraction

Scaffolded Practice 1.1.1

Example 1

Identify each term, coefficient, constant, and factor of $2(3 + x) + x(1 - 4x) + 5$.

1. Simplify the expression.

2. Identify all terms.

3. Identify any factors.

4. Identify all coefficients.

5. Identify any constants.

continued

Example 2

A smartphone is on sale for 25% off its list price. The sale price of the smartphone is $149.25. What expression can be used to represent the list price of the smartphone? Identify each term, coefficient, constant, and factor of the expression described.

Example 3

Helen purchased 3 books from an online bookstore and received a 20% discount. Each book cost the same amount. The shipping cost was $10 and was not discounted. Write an expression that can be used to represent the total amount Helen paid for 3 books plus the shipping cost. Identify each term, coefficient, constant, and factor of the expression described.

Scaffolded Practice 1.1.2

Example 1

A new car loses an average value of $1,800 per year for each of the first six years of ownership. When Nia bought her new car, she paid $25,000. The expression $25{,}000 - 1800y$ represents the current value of the car, where y represents the number of years since Nia bought it. What effect, if any, does the change in the number of years since Nia bought the car have on the original price of the car?

1. Refer to the expression given: $25{,}000 - 1800y$.

2. 25,000 represents the price of the new car.

continued

Example 2

To calculate the perimeter of an isosceles triangle, the expression $2s + b$ is used, where s represents the length of the two congruent sides and b represents the length of the base. What effect, if any, does increasing the length of the congruent sides have on the expression?

Example 3

Money deposited in a bank account earns interest on the initial amount deposited as well as any interest earned as time passes. This compound interest can be described by the expression $P(1 + r)^n$, where P represents the initial amount deposited, r represents the interest rate, and n represents the number of years that pass. How does a change in each variable affect the value of the expression?

Prerequisite Skills

This lesson requires the use of the following skills:

- evaluating expressions using order of operations
- evaluating expressions for a given value
- identifying parts of an expression

Introduction

Algebraic expressions, used to describe various situations, contain variables. It is important to understand how each term of an expression works and how changing the value of variables impacts the resulting quantity.

Key Concepts

- If a situation is described verbally, it is often necessary to first translate each expression into an algebraic expression. This will allow you to see mathematically how each term interacts with the other terms.
- As variables change, it is important to understand that constants will always remain the same. The change in the variable will not change the value of a given constant.
- Similarly, changing the value of a constant will not change terms containing variables.
- It is also important to follow the order of operations, as this will help guide your awareness and understanding of each term.

Common Errors/Misconceptions

- incorrectly translating given verbal expressions

Prerequisite Skills

This lesson requires the use of the following skills:

- applying the order of operations
- creating ratios
- translating verbal sentences into expressions
- solving linear equations

Introduction

Creating equations from context is important since most real-world scenarios do not involve the equations being given. An **equation** is a mathematical sentence that uses an equal sign (=) to show that two quantities are equal. A **quantity** is something that can be compared by assigning a numerical value. In this lesson, contexts will be given and equations must be created from them and then used to solve the problems. Since these problems are all in context, units are essential because without them, the numbers have no meaning.

Key Concepts

- A **linear equation** is an equation that can be written in the form $ax + b = c$, where a, b, and c are rational numbers. Often, the most difficult task in turning a context into an equation is determining what the variable is and how to represent that variable.
- The variables are letters used to represent a value or unknown quantity that can change or vary. Once the equation is determined, solving for the variable is straightforward.
- The **solution** will be the value that makes the equation true.
- In some cases, the solution will need to be converted into different units. Multiplying by a unit rate or a ratio can do this.
- A **unit rate** is a rate per one given unit, and a **rate** is a ratio that compares different kinds of units.
- Use units that make sense, such as when reporting time; for example, if the time is less than 1 hour, report the time in minutes.
- Think about rounding and precision. The more numbers you list to the right of the decimal place, the more precise the number is.
- When using measurement in calculations, only report to the nearest decimal place of the least accurate measurement. See Guided Practice Example 5.

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Creating Equations from Context

1. Read the problem statement first.
2. Reread the scenario and make a list or a table of the known quantities.
3. Read the statement again, identifying the unknown quantity or variable.
4. Create expressions and inequalities from the known quantities and variable(s).
5. Solve the problem.
6. Interpret the solution of the equation in terms of the context of the problem and convert units when appropriate, multiplying by a unit rate.

Common Errors/Misconceptions

- attempting to solve the problem without first reading/understanding the problem statement
- incorrectly setting up the equation
- misidentifying the variable
- forgetting to convert to the correct units
- setting up the ratio inversely when converting units
- reporting too many decimals—greater precision comes with more precise measuring and not with calculations

Prerequisite Skills

This lesson requires the use of the following skills:

- solving simple linear equations
- comparing rational numbers

Introduction

Inequalities are similar to equations in that they are mathematical sentences. They are different in that they are not equal all the time. An inequality has infinite solutions, instead of only having one solution like a linear equation. Setting up the inequalities will follow the same process as setting up the equations did. Solving them will be similar, with two exceptions, which will be described later.

Key Concepts

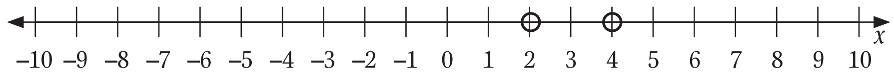
- The prefix *in-* in the word *inequality* means "not." Inequalities are sentences stating that two things are not equal. Remember earlier inequalities such as $12 > 2$ and $1 < 7$.
- Remember that the symbols >, <, ≥, ≤, and ≠ are used with inequalities.
- Use the table below to review the meanings of the inequality symbols and the provided examples with their **solution sets**, or the value or values that make a sentence or statement true.

Symbol	Description	Example	Solution set
>	greater than, more than	$x > 3$	all numbers greater than 3; does not include 3
≥	greater than or equal to, at least	$x \geq 3$	all numbers greater than or equal to 3; includes 3
<	less than	$x < 3$	all numbers less than 3; does not include 3
≤	less than or equal to, no more than	$x \leq 3$	all numbers less than or equal to 3; includes 3
≠	not equal to	$x \neq 3$	includes all numbers except 3

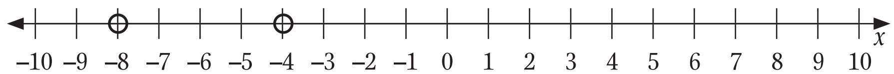
- Solving a linear inequality is similar to solving a linear equation. The processes used to solve inequalities are the same processes that are used to solve equations.

- Multiplying or dividing both sides of an inequality by a negative number requires reversing the inequality symbol. Here is a number line to show the process.
  - First, look at the example of the inequality $2 < 4$.

$2 < 4$

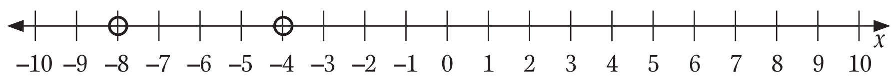


  - Multiply both sides by –2 and the inequality becomes $2(-2) < 4(-2)$ or $-4 < -8$.



  - Is –4 really less than –8?
  - To make the statement true, you must reverse the inequality symbol: $-4 > -8$

$-4 > -8$



Creating Inequalities from Context

1. Read the problem statement first.
2. Reread the scenario and make a list or a table of the known quantities.
3. Read the statement again, identifying the unknown quantity or variable.
4. Create expressions and inequalities from the known quantities and variable(s).
5. Solve the problem.
6. Interpret the solution of the inequality in terms of the context of the problem.

Common Errors/Misconceptions

- not translating the words into the correct symbols, especially with the phrases *no fewer than*, *no more than*, *at least as many*, and so on
- forgetting to switch the inequality symbol when multiplying or dividing by a negative
- not interpreting the solution in terms of the context of the problem—students can often mechanically solve the problem but don't know what the solution means

Scaffolded Practice 1.2.1

Example 1

James earns $15 per hour as a teller at a bank. In one week he pays 17% of his earnings in state and federal taxes. His take-home pay for the week is $460.65. How many hours did James work?

1. Read the statement carefully.

2. Reread the scenario and make a list of the known quantities.

3. Read the statement again and look for the unknown or the variable.

4. Create expressions and inequalities from the known quantities and variable(s).

5. Solve the equation.

6. Convert to the appropriate units if necessary.

continued

Example 2

Brianna has saved $600 to buy a new TV. If the TV she wants costs $1,800 and she saves $20 a week, how many years will it take her to buy the TV?

Example 3

Suppose two brothers who live 55 miles apart decide to have lunch together. To prevent either brother from driving the entire distance, they agree to leave their homes at the same time, drive toward each other, and meet somewhere along the route. The older brother drives cautiously at an average speed of 60 miles per hour. The younger brother drives faster, at an average speed of 70 mph. How long will it take the brothers to meet each other?

Example 4

Think about the following scenarios. In what units should they be reported? Explain the reasoning.

Example 5

Ernesto built a wooden car for a soap box derby. He is painting the top of the car blue and the sides black. He already has enough black paint, but needs to buy blue paint. He needs to know the approximate area of the top of the car to determine the size of the container of blue paint he should buy. He measured the length to be 9 feet $11\frac{1}{4}$ inches, and the width to be $\frac{1}{2}$ inch less than 3 feet. What is the surface area of the top of the car? What is the most accurate area Ernesto can use to buy his paint?

Scaffolded Practice 1.2.2

Example 1

Juan has no more than $50 to spend at the mall. He wants to buy a pair of jeans and some juice. If the sales tax on the jeans is 4% and the juice with tax costs $2, what is the maximum price of jeans Juan can afford?

1. Read the problem statement first.

2. Reread the scenario and make a list or a table of the known quantities.

3. Read the statement again, identifying the unknown quantity or variable.

4. Create expressions and inequalities from the known quantities and variable(s).

5. Solve the problem.

6. Interpret the solution of the inequality in terms of the context of the problem.

continued

Example 2

Alexis is saving to buy a laptop that costs $1,100. So far she has saved $400. She makes $12 an hour babysitting. What's the least number of hours she needs to work in order to reach her goal?

Example 3

A radio station is giving away 40 concert tickets. They give away 1 pair of tickets every hour on the hour for a number of hours until they have at most 4 tickets left for a grand prize. If the contest runs from 11:00 A.M. to 1:00 P.M. each day, for how many days will the contest last?

Prerequisite Skills

This lesson requires the use of the following skills:

- simplifying expressions
- using the distributive property

Introduction

While it may not be efficient to write out the justification for each step when solving equations, it is important to remember that the properties of equality must always apply in order for an equation to remain balanced.

As equations become more complex, it may be helpful to refer to the properties of equality used in the previous lesson.

Key Concepts

- When solving equations, first take a look at the expressions on either side of the equal sign.
- You may need to simplify one or both expressions before you can solve for the unknown. Sometimes you may need to combine like terms by using the associative, commutative, or distributive properties.
- Pay special attention if the same variable appears on either side of the equal sign.
- Just like with numbers, variables may be added or subtracted from both sides of the equation without changing the equality of the statement or the solution to the problem.

Solving Equations with the Variable in Both Expressions of the Equation
1. Choose which side of the equation you would like the variable to appear on. 2. Add or subtract the other variable from both sides of the equation using either the addition or subtraction property of equality. 3. Simplify both expressions. 4. Continue to solve the equation. 5. As with any equation, check that your answer is correct by substituting the value into the original equation to ensure both expressions are equal.

- Some equations may have no solution. This is the case when, after you've completed all of the appropriate steps to solve an equation, the result is something impossible, like $2 = 6$. The resulting equation is never true for any value of the variable.
- Some equations will be true for any value the variable is replaced with. This is the case when following all of the appropriate steps for solving an equation results in the same value on each side of the equal sign, such as $2x = 2x$. The resulting equation is always true for any value of the variable.
- Other equations will only have one solution, where the final step in solving results in the variable equal to a number, such as $x = 5$.

Common Errors/Misconceptions

- performing the wrong operation when isolating the variable
- incorrectly combining terms

Prerequisite Skills

This lesson requires the use of the following skills:

- solving equations
- solving simple inequalities
- applying the distributive property

Introduction

Solving inequalities is similar to solving equations. To find the solution to an inequality, use methods similar to those used in solving equations. In addition to using properties of equality, we will also use properties of inequalities to change inequalities into simpler equivalent inequalities.

Key Concepts

- The **properties of inequality** are the rules that allow you to balance, manipulate, and solve inequalities. The properties are summarized in the following table.

Properties of Inequality

Property	Example
If $a > b$ and $b > c$, then $a > c$.	If $10 > 6$ and $6 > 2$, then $10 > 2$.
If $a > b$, then $b < a$.	If $10 > 6$, then $6 < 10$.
If $a > b$, then $-a < -b$.	If $10 > 6$, then $-10 < -6$.
If $a > b$, then $a \pm c > b \pm c$.	If $10 > 6$, then $10 \pm 2 > 6 \pm 2$.
If $a > b$ and $c > 0$, then $a \bullet c > b \bullet c$.	If $10 > 6$ and $2 > 0$, then $8 \bullet 2 > b \bullet 2$.
If $a > b$ and $c < 0$, then $a \bullet c < b \bullet c$.	If $10 > 6$ and $-1 < 0$, then $10 \bullet -1 < 6 \bullet -1$.
If $a > b$ and $c > 0$, then $a \div c > b \div c$.	If $10 > 6$ and $2 > 0$, then $8 \div 2 > 6 \div 2$.
If $a > b$ and $c < 0$, then $a \div c < b \div c$.	If $10 > 6$ and $-1 < 0$, then $10 \div -1 < 6 \div -1$.

- When solving more complicated inequalities, first simplify the inequality by clearing any parentheses. Do this by either distributing by the leading number or dividing both sides of the inequality by the leading number. Then solve the inequality by following the steps learned earlier, as outlined on the following page.

Solving Inequalities
1. If a variable appears on both sides of the inequality, choose which side of the inequality you would like the variable to appear on.
2. Add or subtract the other variable from both sides of the inequality using either the addition or subtraction property of equality.
3. Simplify both expressions.
4. Continue to solve the inequality as you did in earlier examples by working to isolate the variable.
5. Check that your answer is correct by substituting a value included in your solution statement into the original inequality to ensure a true statement.

- It is important to remember that when solving inequalities, the direction of the inequality symbol (<, >, ≤, or ≥) changes when you divide or multiply by a negative number. Here's an example.
  - If we had the simple statement that $4 < 8$, we know that we can multiply both sides of the inequality by a number, such as 3, and the statement will still be true.

$4 < 8$	Original inequality
$4 \bullet 3 < 8 \bullet 3$	Multiply both expressions of the inequality by 3.
$12 < 24$	This is a true statement.

  - We can also divide both sides of the inequality by a number, such as 2.

$4 < 8$	Original inequality
$4 \div 2 < 8 \div 2$	Divide both expressions of the equation by 2.
$2 < 4$	This is a true statement.

  - Notice what happens when we multiply the inequality by –3.

$4 < 8$	Original inequality
$4 \bullet -3 < 8 \bullet -3$	Multiply both expressions of the inequality by –3.
$-12 < -24$	This is NOT a true statement.

  - To make this a true statement, change the direction of the inequality symbol.

$-12 > -24$	This is a true statement.

- The same is true when dividing by a negative number; you must change the direction of the inequality symbol.

$4 < 8$	Original inequality
$4 \div -2 < 8 \div -2$	Divide both expressions of the equation by –2.
$-2 < -4$	This is NOT a true statement. Change the direction of the inequality symbol.
$-2 > -4$	This is a true statement.

Common Errors/Misconceptions

- not understanding why the inequality symbol changes direction when multiplying or dividing by a negative number
- replacing inequality symbols with equal signs
- forgetting to switch the inequality symbol when multiplying or dividing by a negative number
- switching the inequality symbol when subtracting a number

Scaffolded Practice 3.1.2

Example 1

Solve the equation $5x + 9 = 2x - 36$.

1. Move the variable to one side of the equation.

2. Continue to solve the equation $3x + 9 = -36$.

3. The solution to the equation $5x + 9 = 2x - 36$ is $x = -15$.

continued

Example 2

Solve the equation $7x + 4 = -9x$.

Example 3

Solve the equation $2(3x + 1) = 6x + 14$.

Example 4

Solve the equation $3(4x + 2) = 12x + 6$.

Example 5

Solve the literal equation $A = \frac{1}{2}(b_1 + b_2)h$ for b_1.

Scaffolded Practice 3.1.3

Example 1

Solve the inequality $\frac{-3x-4}{7}>5$.

1. Isolate the variable by eliminating the denominator.

2. Isolate the variable.

3. Divide both sides of the inequality by the coefficient, –3.

4. The solution to the original inequality $\frac{-3x-4}{7}>5$ is all numbers less than –13. To check this, choose any number less than –13 to show a true statement. Let's try –20. Be sure to substitute the value into the original inequality.

continued

Example 2

Solve the inequality $5x + 4 \geq 11 - 2x$.

Prerequisite Skills

This lesson requires the use of the following skills:

- order of operations
- solving multi-step equations

Introduction

Literal equations are equations that involve two or more variables. Sometimes it is useful to rearrange or solve literal equations for a specific variable in order to find a solution to a given problem. In this lesson, literal equations and **formulas**, or literal equations that state specific rules or relationships among quantities, will be examined.

Key Concepts

- It is important to remember that both literal equations and formulas contain an equal sign indicating that both sides of the equation must remain equal.
- Literal equations and formulas can be solved for a specific variable by isolating that variable.
- To isolate the specified variable, use inverse operations. When coefficients are fractions, multiply both sides of the equation by the **reciprocal**. The reciprocal of a number, also known as the **inverse** of a number, can be found by flipping a number. Think of an integer as a fraction with a denominator of 1. To find the reciprocal of the number, flip the fraction. The number 2 can be thought of as the fraction $\frac{2}{1}$. To find the reciprocal, flip the fraction: $\frac{2}{1}$ becomes $\frac{1}{2}$. You can check if you have the correct reciprocal because the product of a number and its reciprocal is always 1.

Common Errors/Misconceptions

- solving for the wrong variable
- improperly isolating the specified variable by using the opposite inverse operation
- incorrectly simplifying terms

Scaffolded Practice 1.5.1

Example 1

Solve $6y - 12x = 18$ for y.

1. Begin isolating y by adding $12x$ to both sides.

2. Divide each term by 6.

continued

Example 2

Solve $15x - 5y = 25$ for y.

Example 3

Solve $4y + 3x = 16$ for y.

Example 4

The formula for finding the area of a triangle is $A = \frac{1}{2}bh$, where b is the length of the base and h is the height of the triangle. Suppose you know the area and height of the triangle, but need to find the length of the base. In this case, solving the formula for b would be helpful.

Example 5

The distance, d, that a train can travel is found by multiplying the rate of speed, r, by the amount of time that it is travelling, t, or $d = rt$. Solve this formula for t to find the amount of time the train will travel given a specific distance and rate of speed.